IMPROVING ON VAR

DelVaR is a new mechanism for assessing the potential impact of a trade on a firm's VAR. Mark Garman explains how it works and the benefits it offers

Analytic variance-covariance value-at-risk is an established technique for measuring exposure to market-based financial risk (Smithson, 1996i and 1996ii). Given a description of the market characteristics and the user's portfolio, the objective of VAR is to determine how much value might be lost over a given time, with a given level of probability, in a given currency. For example, JP Morgan's RiskMetrics provides a comprehensive analytical methodology for assessing VAR.[1]

This form of VAR begins by replacing a portfolio, or the trades within it, with a set of cashflows reflecting those trades' current values and their risk attributes. (This process is sometimes separately referred to as "shredding" the trades.) The resulting cashflows (which can be in any currency, commodity or other price risk source) are then aligned upon certain "vertices" representing standardised maturities and credit levels for the markets in which the cashflows are traded, again preserving their value and risk characteristics. (For example, six-month Libor Deutschmark and two-year dollar swap market flows might each represent a vertex.) The complete process of translating a trade into vertex cashflows is called "mapping". Having thus arrived at the "cashflow map" (ie, the net result of mapping a portfolio of trades), one next combines this map with a covariance matrix whose index set is the set of vertices, which yields the desired result, the VAR number.

Yet after VAR is assessed, there remains an important question: "What can one do to reduce VAR?" In this article, we introduce an approximation mechanism for improving VAR, referred to here as DelVaR.

To present the issues in more concrete terms, consider what feedback the VAR calculation should provide to the trading activity of an institution. Which new trades will improve VAR and which will degrade it? And precisely how will we implement trading limits based upon VAR, when such limits evidently depend not only upon the proposed trades themselves but also on the way in which these trades interact with the existing portfolio of the institution? Unfortunately, the non-linear nature of VAR requires us to create a new portfolio incorporating the proposed trades, and then reassess the VAR of this augmented portfolio. Among other things, this means that the VAR limits imposed upon any proposed new trade necessitate the re-evaluation of all trades (perhaps tens of thousands of these) in the institution's revised portfolio, which can be a demanding process.

The DelVaR approach offers a more economical means for evaluating any new proposed trade in terms of its effect on institutional VAR, without the extensive recalculation of total VAR. This permits the realisation of rapid evaluation of candidate trades, so that real-time VAR trading limits become practical.

Also, by adding one additional feature to the DelVaR mechanism, called "trade normalisation", it becomes possible to determine not just whether certain trades will increase or decrease VAR but, indeed, what relative ranking those trades should enjoy for VAR reduction purposes.

The maths of DelVaR

In this section we discuss the mathematical basis of the DelVaR construct. As a first step, we establish appropriate notation. Suppose that:

- **P** is the existing institutional portfolio of trades;
- $\mathbf{A}_i$ is the portfolio consisting solely of the ith candidate trade, for i = 1,2,. . .,N;
- $\mathbf{p} = \mathbf{m}(\mathbf{P})$ is a (column) vector of cashflow amounts, where $\mathbf{m}(\,)$ is a cashflow mapping function[2], and the index set of the resulting vector is the set of vertices;
- $\mathbf{a}_i = \mathbf{m}(\mathbf{A}_i)$ is a (column) vector of cashflow amounts for the ith candidate trade; and
- **Q** is a variance-covariance matrix scaled by the square of the VAR probability standard deviations (for example, approximately 1.64^2 for the 95% confidence level), the indexes of the matrix also being the vertex index set.

Using the definitions above, the VAR calculation of the portfolio **P** would be given as:

$$\mathrm{VAR} \equiv \upsilon = \sqrt{p'Qp}$$

where the prime means transpose.[3] We now consider the standard approach, namely to perform a new evaluation of the VAR of the augmented portfolio $\mathbf{R}_i = \mathbf{P} + \mathbf{A}_i$ obtained after the ith candidate trade is added to the existing portfolio. The "brute-force" method then requires a complete recalculation of VAR, namely:

$$w_i = \sqrt{r_i' Q r_i}$$

where $\mathbf{r}_i = \mathbf{m}(\mathbf{R}_i)$ is the cashflow map of the augmented portfolio.[4] The test of VAR improvement occurs by examining the difference $(w_i - v)$. When this quantity is negative, the candidate trade will improve institutional VAR; when positive, it will degrade VAR.

Compared with this standard method, the DelVaR approach offers a more economical means of evaluating (approximately) VAR improvement. The mathematical basis of DelVaR is as follows.

Consider the cashflow map vector of a candidate trade, but scaled by the small positive quantity ε, so that VAR is now given by:

$$w_i(\varepsilon) = \sqrt{r_i'(\varepsilon) Q r_i(\varepsilon)}$$

where $\mathbf{r}_i = \mathbf{p} + \varepsilon \mathbf{a}_i$. If we now perform a Taylor series expansion of VAR around $\varepsilon = 0$, we have:

$$w_i(\varepsilon) = w_i(0) + \varepsilon\left[\nabla w_i(0) \cdot a_i\right] + o\left(\varepsilon^2\right)$$
$$= \upsilon + \varepsilon(\mathrm{DelVaR} \cdot a_i) + o\left(\varepsilon^2\right)$$

where V refers to the usual "del" operator, or derivative vector (where the vector index is again vertices), giving rise to the "DelVaR" label. From the latter equation, we see that if ε is sufficiently small (and positive, since we are adding a positive amount of the new candidate trade to our portfolio), the improvement of VAR will be governed by the sign and magnitude of the second term of the last equation above; the higher order terms $(o(\varepsilon^2))$ can reasonably be ignored, provided ε is sufficiently small.

But is ε "sufficiently small" in our context? In most institutions, the answer is normally "yes"; most often, the size of a candidate trade is insignificant relative to the size of the then-current portfolio holdings (as perhaps measured via aggregate cashflow volume). This is

[1] See RiskMetrics – Technical Document, 1995

[2] The cashflow mapping function m( ) embodies the mechanisms which transform cashflows arising in the trades of a portfolio into cashflows located at the vertices, for which volatility, correlation and yield data are then known; see RiskMetrics – Technical Document

[3] Compare with the equation on page 29 of RiskMetrics – Technical Document

[4] A linear cashflow map has the property that m(A + B) = m(A) + m(B) for all trades and portfolios A and B. Note that if linearity of the map function is present, we can speed the process of incorporating the new trade A_i by merging only the mapped cashflows. At present, however, even RiskMetrics contains non-linear mops, eg, for floating-rate notes

"normalised"), then it becomes sensible to assert that one candidate trade or another is to be preferred for the magnitude of its corresponding VAR contribution. The key is properly to define and implement the nature of such alternative comparability.

We suggest six methods of normalisation: cashflow, VAR, return, price, capital and notional. Each method involves dividing each candidate trade's cashflow vector $\mathbf{a}_i$ by the positive scalar number λ_i (its "norm"), where this quantity is calculated in a manner (detailed below) depending upon the trade in question and the method selected. It is of course a matter of risk management policy as to which type of norm is chosen.[5]

Cashflow normalisation (VAR change per unit of cashflow). In this method, some simple mathematical norm for the cashflow vector is associated with each candidate trade. If $\mathbf{a}_i = (a_{i1}, a_{i2}, \ldots, a_{1n})$ is the vector of cashflows, then the norm may be defined by:

$$\lambda_i = \|a_i\| \equiv \sqrt{\sum_j a_{ij}^2}$$

(cashflow length) or

$$\lambda_i = \|a_i\| \equiv \sum |a_{ij}|$$

(sum of absolute cashflow values) or

$$\lambda_i = \|a_i\| \equiv \max_j \{|a_{ij}|\}$$

(maximal cashflow). Each of these sub-cases may provide a slightly different result, but all are dictated merely by the sizes of the mapped cashflows which correspond to a candidate trade.

VAR normalisation (VAR change per unit of trade VAR). In this method, the scaling is performed according to the VAR inherent in the candidate trade itself. (In effect, each candidate trade is evaluated on the basis of equating the risk – as measured via VAR – as if the candidate trade were held in isolation.) Accordingly, the norm is then calculated as:

$$\lambda_i \equiv \sqrt{a_i' Q a_i}$$

where $\mathbf{Q}$ is the scaled variance-covariance matrix described more fully in the previous section.

Return normalisation (VAR change per unit of return). In this case, the norm λ_i is selected according to the anticipated future returns accruing to the investment in the candidate trade. For example, one might cumulate the net present value of all future revenues and payments of the candidate trade, as one such measure of future economic value.[6] Closely related is:

- **Price normalisation** (VAR change per unit of price). Here, λ_i is set equal to the market price of the candidate trade. This equates candidate trades according to their current mark-to-market, ie, value by present market standards.
- **Capital normalisation** (VAR change per unit of capital). In this approach, λ_i is set equal to the regulatory or other amount of capital which must be allocated to sustain that trade. For example, the Bank for International Settlements guidelines provide formulas involving certain amounts of capital underlying certain trade types.
- **Notional normalisation** (VAR change per arbitrary unit of trade). In this approach, we employ the "notional value", ie, an otherwise arbitrary market or other convention on the number of units involved in the candidate trade. For example, swap contracts are typically denominated in amounts involving $1 of principal payment, regardless of the swap interest rates involved. Because this norm is completely arbitrary, it serves as a catch-all category for trade normalisation.

The importance of normalisation is that it permits a ranking of the relative risk-reducing qualities of various candidate trades. By removing the "size" issue through normalisation, trades now become subject to comparison among themselves, for risk reduction purposes. Whatever norm is selected, the ranking is then based upon the comparison of the quantities:

$$\text{DelVaR} \cdot (a_i \;/\, \lambda_i) = [p'Q \,/\, \upsilon] \cdot (a_i / \lambda_i)$$

Again, we note that the same DelVaR quantity is used throughout, regardless of the (normalised) candidate trade.

Implementation

We now provide a more complete view of the preferred means of implementing the DelVaR and normalisation processes. If, say, a daily VAR cycle is selected, then the DelVaR and VAR computations may be constructed at the beginning of the day (perhaps via overnight aggregation and calculation) by analysing the complete portfolio of the financial institution. (Note that there is only a very small marginal effort required to create and store the DelVaR vector in the process of computing VAR, as shown in the figure opposite.)

Subsequently, the VAR limit computation of any proposed trade during the day is done as per the right-hand side of the figure: first, the cashflow map of the proposed trade is constructed; second, ideally (but optionally), the trade is normalised using one of the methods described above; and third, the inner product of the proposed trade's cashflow map and DelVaR is computed. This latter quantity discloses whether the corresponding trade is VAR-increasing or VAR-decreasing and, if properly normalised, also makes a meaningful statement about the relative magnitude of such VAR augmentation.

References

RiskMetrics - Technical Document, third edition, 1995, JP Morgan

Smithson, C, 1996i, *Value-at-risk*, *Risk* January 1996, pages 25-27

Smithson, C, 1996ii, *Value-at-risk (2)*, *Risk* February 1996, pages 38-39

Conclusion

The DelVaR and normalisation processes provide a way of determining the signs and relative magnitudes, respectively, of changes to portfolio VAR resulting from proposed trades. The inner product of DelVaR and the cashflow map of any proposed normalised trade provides a good approximation of the *per-unit* impact that a proposed trade will have, without necessitating a full recalculation of augmented portfolio VAR. DelVaR is therefore useful for implementing real-time VAR trading limits and other related calculations which must rapidly assess the risk management impact of a proposed trade. As a result of using the DelVaR concept, the effect of VAR technology upon trading activities becomes much more direct, immediate and informative. ■

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[5] This policy issue can be addressed by asking "What is the scarcest resource being employed in a trade?" If this is "lines", then perhaps cashflow normalisation might be indicated; if it is capital, certainly capital normalisation should be considered

[6] To be a valid norm, this must always be positive